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Dynamic Spectrum Auction in Wireless Communication

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Preface

Static spectrum allocation hinders the efficient use of spectrum, one of the most valuable and fundamental resources for wireless communication. Spectrum auction, which enables new users to gain spectrum access and existing spectrum owners to obtain financial benefits, can greatly improve spectrum efficiency by resolving the problem of artificial spectrum shortage. However, spectrum auction design faces significant challenges due to the nature of the spectrum, including reusability, spatial and temporal availability. This book focuses on the state-of-art research on spectrum auction design, including fundamental auction theory, characteristics of spectrum market, spectrum auction architecture and possible auction mechanisms.

Hong Kong — Yanjiao Chen
August 2014 — Qian Zhang

Contents

Acronyms

dB	Decibel
FCC	Federal Communications Commission
GHz	Gigahertz
IID	Independent and Identically Distributed
ITU	International Telecommunication Union
MHz	Megahertz
PCAST	President's Council of Advisors on Science and Technology
SMRA	Simultaneous Multiple Round Auction
VCG	Vickrey-Clarke-Groves

Chapter 1
Introduction

Spectrums are indispensable resources for wireless communication [34]. Propelled by the rapid development of smart devices and 4G technology, the demand for wireless traffic increases exponentially. In 2010, users worldwide downloaded 5 billion mobile applications, 15 times more than the figure (300 million) in 2009. In the U.S., the number of subscribers to mobile services increased by 20 million in 2011 alone, amounting to 294 million [3]. Such a demand will surpass the capacity of allocated wireless spectrums for mobile broadband services by as soon as 2013 [55]. To deal with this problem, on the one hand, the regulators are releasing more spectrums for commercial use; on the other hand, secondary spectrum markets emerge where incumbent spectrum licensees lease their spectrums to other service providers. In 2010, the Federal Communications Commission (FCC) in the U.S. decided to make 500 MHz of new wireless spectrum available within ten years [54]. In July 2012, the President's Council of Advisors on Science and Technology (PCAST) of the U.S. further proposed to identify 1000 MHz of federal spectrum for commercial use [51]. In 2010, the FCC introduced the idea of incentive auction to encourage incumbent spectrum licensees to voluntarily give up their license and get part of the revenue from re-selling their spectrums [3]. Company Spectrum® Bridge has launched an online platform called SpecEx for spectrum owners to sell their unused spectrums to potential buyers [1]. Spectrum auction can be an efficient way to reallocate these spectrums, either from the regulators to the wireless service providers or from incumbent spectrum licensees to secondary service providers [17].

Spectrum auction is different from traditional auction mainly due to the nature of spectrums, especially the reusability characteristic [28, 61]. A spectrum can be reused by multiple buyers if they don't interfere with each other[1] [30, 38]. Because of path loss, the transmission range of a signal is limited [4, 20]. If buyer A is beyond the transmission range of buyer B, then buyer B's transmission will not affect buyer A. The transmission range of a spectrum depends on its central frequency. By

[1] We assume that the entire available spectrum band are divided into spectrums with equal bandwidth. Therefore, we refer to "spectrum" as countable commodities.

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leveraging reusability, a spectrum can be auctioned to multiple buyers, as long as interference constraints are obeyed. This can greatly improve spectrum utilization, but poses challenges for auction design. One of the fundamental requirements for auction design is truthfulness, which means that any buyer or seller will bid their true valuations for the auctioned commodities [41, 47]. However, traditional auction mechanisms, when applied directly to spectrum auction, will become untruthful [71, 72]. In other words, auction participants have opportunities to manipulate their bids to gain higher utilities, which disrupts the economic robustness of the auction. Therefore, new auction mechanisms are need to address the spectrum reusability while maintaining nice economic properties.

Apart from spectrum reusability and economic properties, there are four other concerns in the spectrum auction design.

- *Auction Format*. Forward auction, reverse auction or double auction.
- *Demand/supply restrict*. Single item auction or multiple item auction.
- *Spectrum attribute.* Homogeneous spectrums or heterogeneous spectrums.
- *Auction dynamics.* Static auction or dynamic auction (also known as online auction).

In the forward auction, there is one seller and multiple buyers; in the reverse auction, there is one buyer and multiple sellers; in the double auction, there are multiple sellers, multiple buyers and one auctioneer. The auctioneer takes the responsibility of collecting asks from the sellers and bids from the buyers, deciding the spectrum allocation and the prices. Forward auction and double auction are the most common spectrum auction formats while reverse auction is seldom used because in common cases, there are more spectrum demands than spectrum supplies. In single item auction, each seller or buyer is restricted to trade one spectrum; while in multiple item auction, each seller or buyer is allowed to trade multiple spectrums. Multiple item auction is more flexible than single item auction, but more difficult to ensure truthfulness.

If spectrums are treated as homogeneous, there is no distinction between spectrums with different central frequencies. If spectrum heterogeneity is considered, several issues will arise. First, buyers and sellers may have different valuations for different spectrums. A spectrum with long transmission range may be suitable for large cell size (e.g. macrocell network); while a spectrum with short transmission range may be desirable for small cell size (e.g. femtocell network). The interference relationship between buyers will become quite complicated. If a buyer's device operates on a high frequency spectrum, he will interfere with a shorter range of other buyers; if a buyer's device operates on a low frequency spectrum, he will interfere with a wider range of other buyers. To decide which buyers can reuse the same spectrum becomes challenging.

In the static spectrum auction, the auction only lasts for one time stage. Static spectrum auction is suitable for long-term spectrum allocation, where the spectrum availability, the wireless environment and the interference relationship are relatively stable. In the dynamic spectrum auction, the auction will be performed for finite or infinite time stages. Dynamic spectrum auction is quite different from static spectrum auction. In the dynamic spectrum auction, the buyers may come sporadically, and the auction results in the earlier time stages will affect those in the latter time stages. For example, if a spectrum is allocated to buyer A for 2 time slots in the first stage, it cannot be allocated to other buyers who interfere with buyer A in the second stage. This makes it difficult to decide how to allocate spectrums in every time stage. To solve this problem, it is needed to estimate the influence of current spectrum allocation on the spectrum allocation in the following time stages.

In this book, we mainly focus on sealed-bid, collusion-free auction. Sealed-bid means that all bidders simultaneously submit their bids, so that no bidder knows the bids of any other bidders. Collusion-free means that no bidders collude with each other to improve the utility of the collusion group. In Chap. 5, we will discuss the problem of collusion in the spectrum auction as a future research direction. In the rest of this chapter, we will describe the background of spectrum auction in more details. In Chap. 2, we will introduce static spectrum auction mechanisms which treat spectrums as homogeneous commodities, in both forward and double auction formats. In Chap. 3, we will consider spectrum heterogeneity and introduce a static heterogeneous spectrum double auction mechanism. In Chap. 4, we will focus on online spectrum auction and introduce a dynamic heterogeneous spectrum double auction mechanism. Finally, in Chap. 5, we will give future research directions on spectrum auction.

1.1 Property of Spectrums

In this section, we will show the basic transmission model, based on which the spectrum reusability is determined.

1.1.1 Transmission Range and Spectrum Reusability

The transmission range of a spectrum determines the interference relationship among buyers, which is important for determining spectrum reusability. The power of an electromagnetic wave will decrease as it propagates through free space. The reduction of the power is usually referred to as path loss. Path loss is influenced by the environment (urban or rural), propagation medium (humidity of the air), the distance between the transmitter and the receiver, the location of the antenna, and the central frequency of the spectrum. According to the propagation model recommended by the International Telecommunication Union (ITU) [52], the path loss is affected by the central frequency of a spectrum according to the following function.

$$L = 10 \log f^2 + \gamma \log d + P_f(n) - 28 \tag{1.1}$$

in which L is the total path loss in decibel (dB), f is the central frequency of the spectrum in megahertz (MHz), d is the transmission distance in meter (m), γ is the distance power loss coefficient and $P_f(n)$ is the floor loss penetration factor. Let P_t and P_r denote the transmission power and targeted receiving power, respectively. The maximum allowable path loss is $L_{max} = P_t - P_r$. Therefore, the maximum transmission range is:

$$R_{max} = \exp\left\{\frac{P_t - P_r + 28 - P_f(n) - 10\log f^2}{\gamma}\right\} \tag{1.2}$$

Given the central frequency of a spectrum, its transmission range R_{max} can be computed by (1.2). It is obvious that, a high frequency spectrum with a larger f has a shorter transmission range, while a low frequency spectrum with a smaller f has a longer transmission range.

Assume that a transmitter operates on a spectrum with central frequency f and transmission range R_{max} determined by (1.2), other user devices within the range of R_{max} will be interfered. The interference relationship between two users is often not symmetric, even if they operate on the same spectrum [50, 59]. This is because the channel conditions between the two users is often asymmetric, i.e., the $P_f(n)$ and γ are different in (1.2).

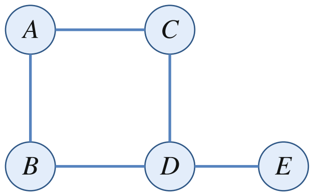

Fig. 1.1. Illustration of the interference graph

1.1.2 Interference Graph

Interference graph is the most common method to represent interference relationship among buyers[2]. It is an undirected graph constructed based on the transmission range of the spectrum and geographic information of the buyers [11, 56]. Therefore, interference graph is spectrum-specific. In other words, different spectrums with different central frequencies should have different interference graphs since their transmission ranges are different. Interference graph makes it easy to apply graph theory to solve the problem of spectrum reusability. Let $G = (V, E)$ denote an interference graph based on a specific spectrum. V is the set of nodes, and E is the set of edges. Each node represents a buyer. If two buyers interfere with each other, there is an edge between them; otherwise, there is no edge between them. Since

[2] Some works also used interference temperature instead of interference graph [24, 68].

the interference graph is undirected, it is implicitly assumed that the interference relationship between any two buyers is symmetric. Two nodes without an edge between them can reuse the same spectrum. For example, nodes A and D in Fig. 1.1. Furthermore, a group of nodes that share no edges can reuse the same spectrum. For example, nodes B, C and E in Fig. 1.1. To find such group of nodes is equivalent to finding an independent set on the interference graph, a classic problem in graph theory with many ready-to-use algorithms [6, 10, 44].

1.2 Traditional Auction Mechanisms

In this section, we briefly introduce three well-known truthful auction mechanisms. The major drawback of these auction mechanisms is that they don't consider spectrum reusability.

1.2.1 Secondary Auction

We take multiple item forward auction as an example. Secondary auction mechanism processes as follows. First, sort the buyers' bids in non-ascending order. If there are M items, name the top M buyers as winners and charge them the $(M + 1)$th buyer's bid. A simple extension of secondary auction to forward spectrum auction is shown to be untruthful in [71].

1.2.2 Vickrey-Clarke-Groves Auction

Vickrey-Clarke-Groves (VCG) auction mechanism tries to maximize social welfare with feasible allocation [16, 29, 62]. Feasible allocation refers to the auction results that satisfy the constraints of the auction (e.g., total number of auctioned items). We take forward auction as an example. Social welfare is defined as the total valuation of all the winning buyers [48]. We presume that the auction is truthful, so that the total valuation equals the total bid of all the winning buyers. Let b_i denote the bid of buyer i. First, find one optimal feasible allocation A^* that maximizes the total bid of all winning buyers (usually through brute force). For a winning buyer i, assume that in the optimal allocation, all the other buyers gain utility $\sum_{j \neq i} b_j(A^*)$. Having removed buyer i, we can find another optimal feasible allocation $\widetilde{A}^*$, all the buyers except i will gain utility $\sum_{j \neq i} b_j(\widetilde{A}^*)$. Then buyer i will be charged the price $\sum_{j \neq i} b_j(\widetilde{A}^*) - \sum_{j \neq i} b_j(A^*)$. Although VCG mechanism possesses many good properties such as truthfulness, the computational complexity is its major drawback. Approximate-VCG mechanisms have been explored [40, 42] to achieve polynomial time complexity while maintain truthfulness or approximate truthfulness. VCG auction mechanism can be proved to be truthful for traditional auction. However, a simple extension of VCG auction mechanism to forward spectrum auction is shown to be untruthful in [71], and a simple extension of VCG to double spectrum auction is shown to violate the economic property of budget balance in [72].

1.2.3 McAfee Auction

Single-item double auction [7, 18] and multi-item double auction [8, 32] mechanisms have been developed, mainly following the idea of McAfee [45]. Assume that each buyer or seller has one item to trade. First, sort the sellers' asks in non-descending order and sort the buyers' bids in non-ascending order (sellers' bidding prices are often referred to as "asks"; while buyers' bidding prices are often referred to as "bids"). Then, find index k so that the kth seller's ask is no greater than the kth buyer's bid, but the $(k+1)$th seller's ask is strictly greater than the $(k+1)$th buyer's bid. After doing so, the first $(k-1)$ buyers and $(k-1)$ sellers become winners. Each winning seller is paid by the kth seller's ask; and each winning buyer pays by the kth buyer's bid. The static homogeneous spectrum double auction we introduce in Chapter 2 follows the design rationale of McAfee, but carefully design the spectrum allocation and pricing mechanisms to enable spectrum reusability and guarantee truthfulness.

1.3 Economic Properties

In this section, we introduce three economic properties that are deemed to be most essential for spectrum auction design.

1.3.1 Truthfulness

Truthfulness is one of the most fundamental property of an auction mechanism [39]. The buyers and sellers are selfish and rational players, who will manipulate their asks and bids to maximize their own utilities. Being truthful means that a seller's ask or a buyer's bid equal their true valuations for the spectrum[3]. A truthful auction mechanism guarantees that a buyer or a seller cannot get higher payoff by misreporting their true valuations, thus they will have no incentive to be untruthful. For the online spectrum auction, we have to further consider truthfulness at each time stage.

[3] A broader meaning of truthfulness may also include that a buyer truthfully reports his spectrum demand or time slot requirement.

1.3.2 Individual Rationality

A buyer or a seller is individually rational in the sense that they will not participate in the auction, if by doing so their utilities become negative. An auction mechanism is individually rational, if all sellers and buyers achieve non-negative utility. In other words, in an individual rational auction, any seller is paid more than his ask, and any buyer pays less than his bid.

1.3.3 Budget Balance

Budget balance is often considered in the double auction. It means that the auctioneer maintains non-negative budget. In other words, the money that the auctioneer gets from all buyers is no less than the money he gives to all sellers. For regulators, budget balance is often enough to motivate them to host spectrum auctions. The profit-oriented auctioneers, however, may aim at revenue maximization.

Ideally, we want the auction mechanisms to possess the above three economic properties while maximizing spectrum utilization via spectrum reuse. However, it has been proved that no double auction mechanism can achieve highest spectrum utilization and maintain economic properties at the same time [49, 69]. Most of the auction mechanisms target at economic robustness [7, 8, 18, 32, 72]. Some spectrum auction mechanisms also aim at revenue maximization [5, 25, 53] or collusion resistance [66–68].

Chapter 2
Static Homogeneous Spectrum Auction

In this chapter, we will introduce static homogeneous spectrum auction, first in forward auction format, then in double auction format. The key of the auction design is to leverage spectrum reusability while guaranteeing economic properties.

2.1 Homogeneous Spectrum Forward Auction

In the homogeneous spectrum forward auction, we assume that there is one seller who owns k spectrums, and there are N buyers. Buyer i's bid consists of two parts: d_i is the number of spectrums wanted and b_i is the bidding price. In homogeneous spectrum auction, buyer i's bidding price is the same for all spectrums. For strict request, buyer i either accepts d_i spectrums or 0 spectrum; for range request, buyer i accepts any spectrums in the range $[0, d_i]$. Here, we consider strict request. The bid of a buyer is based on his true valuation for the spectrum, denoted by v_i. In the homogeneous spectrum auction, buyer i has the same valuation for all spectrums. If buyer i becomes a winner, the seller will charge him p_i for each spectrum; otherwise, the seller will charge him nothing. Buyer i's utility is his valuation for the obtained spectrum minus the paid price.

$$U_i = \begin{cases} (p_i - v_i) * d_i, & \text{if buyer } i \text{ wins} \\ 0, & \text{otherwise} \end{cases} \tag{2.1}$$

2.1.1 *A Naive Truthful Auction Mechanism*

We first introduce a naive auction mechanism based on secondary auction, which is truthful but greatly reduces spectrum utilization. As shown in Fig. 2.1, suppose that all the buyers are located in a rectangular region. The maximum transmission range of all the spectrums is R_{max}. We partition the whole region into small squares

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of $R_{max} \times R_{max}$ area. Therefore, buyers in two non-adjacent squares do not interfere with each other. We divide the spectrums into four sets, each set containing $k/4$ spectrums. In each square, we apply secondary auction, allocating $k/4$ spectrums to the top $k/4$ bidders within the square and charge them the price of the $(k/4+1)$th highest bid. Every two adjacent squares will have different sets of spectrums. In this way, the spectrum allocation will be interference free, and the spectrums can be partially reused. However, not every two buyers in a square interfere with each other. Since only $1/4$ of the total spectrums are allocated in each square, the spectrum utilization will be low.

2.1.2 Auction Mechanism Based on Greedy Algorithm

To improve spectrum utilization, an auction mechanism based on greedy algorithm VERITAS is proposed in [71]. The major algorithm includes two parts: spectrum allocation and price determination.

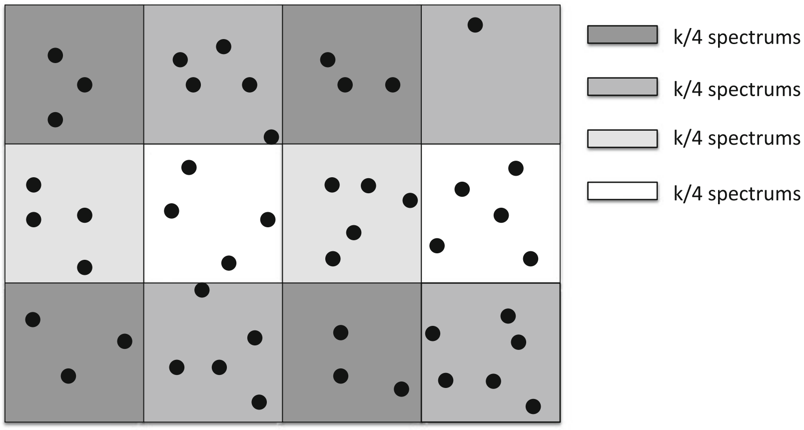

Fig. 2.1. A naive truthful auction mechanism

Spectrum Allocation Algorithm 1 shows the procedure of spectrum allocation. To begin with, the buyers are sorted according to their bids in non-ascending order. In each iteration, the unassigned buyer with the highest bid is considered. Let $N(i)$ denote the set of interfering neighbors of buyer i. If his spectrum demand d_i is fewer than the number of available spectrums, that is, k minus the spectrums assigned to his interfering neighbors $\sum_{j \in N(i)} d_j$, his demand can be fulfilled; otherwise, he will not be assigned any spectrums because of strict request.

Price Determination Algorithm 2 is the procedure of price determination. The idea of price determination is to charge a buyer the unit price which equals the bid of his critical neighbor.

Definition 1 *Critical neighbor*. The critical neighbor of buyer i is one of his interfering neighbors $j \in N(i)$. If buyer i bids no smaller than buyer j, that is, $b_i \geq b_j$, buyer i will be allocated d_i spectrums; otherwise, buyer i will be allocated no spectrums.

Algorithm 1 Spectrum allocation in VERITAS

$B =$ Sorted list of buyers according to their bids in non-ascending order.
while $B \neq \Phi$ **do**
  i is the first buyer in B.
  if $d_i \leq M - \sum_{j \in N(i)} d_j$ **then**
    Assign d_i spectrums to buyer i.
  end if
  Remove buyer i from B.
end while

Algorithm 2 shows how to find the critical neighbor for a buyer in an efficient way. To calculate the price for buyer i, firstly, buyer i is removed from the sorted buyer list B. Then, the algorithm runs like Algorithm 1, allocating spectrums to buyers iteratively. Every time an interfering neighbor j of buyer i is allocated spectrums, we check whether the rest of the spectrums is sufficient for buyer i. If not, buyer j is the critical neighbor of buyer i. This is because if buyer i bids lower than buyer j, he will be placed behind buyer j in the sorted list B. When it comes to buyer i's iteration, there will not be enough spectrums for him, and he will be allocated no spectrums.

Algorithm 2 Price determination for buyer i in VERITAS

$p_i = 0$.
if $d_\mathrm{i} = 0$ **then**
  Return.
end if
$B' = B \backslash \{b_i\}$.
$Avail = k$.
while $B' \neq \Phi$ AND $flag = 1$ **do**
  j is the first buyer in B'.
  if $d_j \leq M - \sum_{l \in N(j)} d_l$ **then**
    Assign d_j spectrums to buyer j.
    if $j \in N(i)$ **then**
      $Avail = Avail - d_j$.
      if $Avail < d_i$ **then**
        $p_i = b_j$.
        $flag = 0$.
      end if
    end if
  end if
  Remove buyer j from B'.
end while

2.1.3 Proofs of Economic Properties

Individual Rationality To prove that the buyers have non-negative utility, we only have to prove the following proposition.

Proposition 1 *The unit price charged from a buyer is always smaller than his bid.*

Proof If $p_i = 0$, it is clear that $p_i \leq b_i$. If $p_i > 0$, it means that buyer i is allocated spectrums and p_i is the bid of his critical bidder. According to Definition 1 of the critical bidder, $b_i \geq p_i$, otherwise buyer i cannot win any spectrums. Therefore, we have proved that the unit price charged from a buyer is always smaller than his bid.

Truthfulness To prove the truthfulness of the auction, we only have to prove the following proposition.

Table 2.1 Possible auction results

Case	I	II	III	IV
The seller/buyer is truthful	Lose	Win	Win	Lose
The seller/buyer is untruthful	Lose	Lose	Win	Win

Proposition 2 *Any buyer cannot gain higher utility when he bids untruthfully than when he bids truthfully.*

Proof When a buyer i bids truthfully and untruthfully, possible auction results are listed in Table 2.1. Let U_i and p_i denote buyer i's utility and price when he bids truthfully; U_i' and p_i' denote buyer i's utility and price when he bids untruthfully; v_i denote buyer i's true valuation for each spectrum. We prove that in every case, $U_i \geq U_i'$.

- Case I. As buyer i loses when he bids truthfully and untruthfully, $U_i = U_i' = 0$.
- Case II. As buyer i loses when he bids untruthfully, $U_i' = 0$. As buyer i wins when he bids truthfully, $U_i \geq 0$ according to individual rationality. Therefore, $U_i \geq U_i'$.
- Case III. As buyer i wins when he bids truthfully and untruthfully, p_i and p_i' both equal the bid of his critical neighbor, therefore $U_i = U_i'$.
- Case IV. As buyer i loses when he bids truthfully, $U_i = 0$. Also, this means that his bid, which equals his true valuation v_i, is smaller than the bid of his critical neighbor. When buyer i wins by bidding untruthfully, it must be true that $p_i' > v_i$ because p_i' equals the bid of his critical neighbor. Therefore we have $U_i' = (v_i - p_i') * d_i < 0 = U_i$.

In summary, we have proved that $U_i \geq U_i'$ for all possible auction results. Therefore, a buyer has no incentive to bid untruthfully in the auction.

2.2 Homogeneous Spectrum Double Auction

In the homogeneous spectrum double auction, we assume that there are M sellers and N buyers, and that each seller owns one spectrum and each buyer wants to buy one spectrum. We assume that the spectrums are available to all buyers. Let b_i^s, v_i^s, U_i^s denote the ask, true valuation and utility of seller i; b_j^b, v_j^b, U_j^b denote the bid, true valuation and utility of buyer j; U^a denote the utility of the auctioneer. After the auction, the auctioneer pays seller i price p_i^s and charges buyer j price p_j^b. The seller i's utility is his payment minus his true valuation for the spectrum.

$$U_i^s = \begin{cases} p_i^s - v_i^s, & \text{if seller } i \text{ wins} \\ 0, & \text{otherwise} \end{cases} \tag{2.2}$$

The buyer j's utility is his true valuation for the spectrum minus his price.

$$U_j^b = \begin{cases} v_j^b - p_j^b, & \text{if buyer } j \text{ wins} \\ 0, & \text{otherwise.} \end{cases} \tag{2.3}$$

The auctioneer's utility is his collected payment from all buyers minus his payment to all sellers.

$$U^a = \sum_j p_j^b - \sum_i p_i^s. \tag{2.4}$$

2.2.1 Auction Mechanism Design

In this section, we introduce a truthful spectrum double auction mechanism TRUST [72, 73], which consists of three processes: grouping, spectrum allocation, and price determination.

Grouping The auctioneer groups the buyers who can reuse the same spectrum together. To find such buyer groups, the auctioneer can construct interference graph of all buyers, then find independent sets on the graph. To avoid market manipulation, the grouping process is performed privately by the auctioneer, unknown by neither the sellers or the buyers. Assume there are L buyer groups. Let $g_1, g_2, ..., g_L$ denote the resulting groups, and $|g_i|$ is the number of buyers in group g_i. If g_i is allocated a spectrum, all the members in g_i will reuse the spectrum.

Spectrum Allocation Algorithm 3 shows the process of spectrum allocation. First, the bid of a group is calculated based on the lowest bid in the group. Then, the sellers are sorted according to their asks in non-descending order, and the buyer groups are sorted according to their group bids in non-ascending order. The last profitable trade is the kth one in which the kth buyer group's bid is no less than the kth seller's ask, but the $(k+1)$th buyer group's bid is strictly smaller than the $(k+1)$th seller's ask. The first $(k-1)$ sellers and $(k-1)$ buyer groups are the winners. Since the spectrums are homogeneous, we can randomly assign the $(k-1)$ spectrums to the $(k-1)$ winning groups.

Algorithm 3 spectrum allocation in TRUST

1: Calculate the group bid of g_i as

$$B_i = |g_i| \times \min_{j \in g_i} b_j^b \tag{2.5}$$

2: Sort the sellers according to their asks in non-descending order.
3: Sort the buyer groups according to their bids in non-ascending order.
4: Find k that $B_k \geq b_k^s$ and $B_{k+1} < b_{k+1}^s$.
5: The auction winners are the top $k-1$ sellers and the buyers in the top $k-1$ groups.
6: Randomly allocate the spectrums of the top $k-1$ sellers to the top $k-1$ buyer groups.

Price Determination If a seller or a buyer is not a winner, he will be paid or pay nothing. If seller i is among the top $k-1$ sellers, the auctioneer will pay seller i the price of the kth seller.

$$p_i^s = b_k^s. \tag{2.6}$$

If buyer group g_j is among the top $k-1$ buyer groups, the auctioneer will charge buyer group g_j the price of the kth buyer group, which will shared by all members in the group.

$$p_l^b = B_k/|g_j|, \forall l \in g_j. \tag{2.7}$$

2.2.2 *Proofs of Economic Properties*

Individual Rationality

Proposition 3 *The auction mechanism TRUST is individual rational for both buyers and sellers.*

Proof If a buyer or a seller is not a winner, his utility is zero (non-negative).

If seller i is a winner, he is paid $p_i^s = b_k^s \geq b_i^s$ since the sellers are sorted in non-descending order.

If buyer l is a winner in group g_i, he pays $p_l^b = B_k/|g_i| \leq B_i/|g_i| = \min_{j \in g_i} b_j^b \leq b_l^b$.

Budget Balance

Proposition 4 *The auctioneer's utility is non-negative.*

Proof The auctioneer's utility is as follows.

$$U^a = (k-1)(B_k - b_k^s) \geq 0$$

Truthfulness We have to prove truthfulness both on buyers' side and on sellers' side. According to Table 2.1, there are four possible auction results.

Proposition 5 *The auction mechanism is truthful for sellers.*

Proof Let U_i^s and p_i^s denote seller i's utility and price when he bids truthfully; and $U_i^{s'}$ and $p_i^{s'}$ denote seller i's utility and price when he bids untruthfully; v_i^s denote seller i's true valuation for the spectrum. We prove that in every case, $U_i^s \geq U_i^{s'}$.

- Case I. As seller i loses when he bids truthfully and untruthfully, $U_i^s = U_i^{s'} = 0$.
- Case II. As seller i loses when he bids untruthfully, $U_i^{s'} = 0$. As seller i wins when he bids truthfully, $U_i^s \geq 0$ according to individual rationality. Therefore, $U_i^s \geq U_i^{s'}$.
- Case III. We first prove that if seller i wins when he bids truthfully and untruthfully, his payments p_i^s and $p_i^{s'}$ are the same.
  When seller i bids truthfully, the sorted seller list is $\{b_1^s, ..., b_k^s, b_{k+1}^s, ...\}$, and the sorted buyer group list is $\{B_1, ..., B_k, B_{k+1}, ...\}$. We have $b_k^s \leq B_k$ and $b_{k+1}^s > B_{k+1}$. When seller i bids untruthfully, it can be easily proved that his bid cannot be greater than b_k^s, otherwise seller i cannot be a winner. Therefore, the top $k-1$ seller and the kth seller do not change. So we have $p_i^s = p_i^{s'} = b_k^s$, and $U_i^s = p_i^s - v_i^s = p_i^{s'} - v_i^s = U_i^{s'}$.
- Case IV. As seller i loses when he bids truthfully, $U_i^s = 0$. The sorted seller list is $\{b_1^s, ..., b_k^s, b_{k+1}^s, ..., b_i^s, ...\}$, and the sorted buyer group list is $\{B_1, ..., B_k, B_{k+1}, ...\}$. We have $b_k^s \leq B_k$ and $b_{k+1}^s > B_{k+1}$. Since seller i loses, we know that $b_i^s \geq b_k^s$. If seller i wins by bidding untruthfully, it must be true that $b_i^{s'} < b_k^s$. The sorted seller list becomes $\{b_1^s, ..., b_i^{s'}, ..., b_{k-1}^s, b_k^s, ...\}$, and the sorted buyer group list does not change. B_k and b_{k-1} become a pair. As $b_{k-1}^s \leq b_k^s \leq B_k$, the final payment for all sellers will be no greater than b_{k-1}^s. Therefore, $p_i^{s'} \leq b_{k-1}^s \leq b_k^s \leq b_i^s = v_i^s$. Hence, $U_i^{s'} = p_i^{s'} - v_i^s \leq 0 = U_i^s$.

Proposition 6 *The auction mechanism is truthful for buyers.*

Proof Let U_i^b and p_i^b denote buyer i's utility and price when he bids truthfully; and $U_i^{b'}$ and $p_i^{b'}$ denote buyer i's utility and price when he bids untruthfully. We prove that in every case, $U_i^b \geq U_i^{b'}$.

- Case I. Similar to seller's CASE I.
- Case II. Similar to seller's CASE II.
- Case III. We first prove that if buyer i wins when he bids truthfully and untruthfully, his price p_i^b and $p_i^{b'}$ are the same.
  When buyer i bids truthfully, the sorted seller list is $\{b_1^s, ..., b_k^s, b_{k+1}^s, ...\}$, and the sorted buyer group list is $\{B_1, ..., B_k, B_{k+1}, ...\}$. We have $b_k^s \leq B_k$ and $b_{k+1}^s > B_{k+1}$.

When seller i bids untruthfully, it can be easily proved that his group's bid cannot be smaller than B_k, otherwise, his group cannot be a winner. Therefore, the top $k-1$ buyer groups and the kth buyer group do not change. So we have $p_i^b = p_i^{b'} = B_k/|g_j|, i \in g_j$, and $U_i^b = p_i^b - v_i^b = p_i^{b'} - v_i^b = U_i^{b'}$.

- Case IV. When buyer i bids truthfully, the sorted seller list is $\{b_1^s, ..., b_k^s, b_{k+1}^s, ...\}$, and the sorted buyer group list is $\{B_1, ..., B_k, B_{k+1}, ..., B_j, ...\}$. Since buyer i loses, we know that $B_j \le B_k$. We have $U_i^b = 0$. As buyer i loses by bidding truthfully and wins by bidding untruthfully, it must be true that b_i^b is the minimum in group g_j, and $b_i^{b'} > b_i^b$ to change the group bid so that $B_j' > B_k$. When buyer i bids untruthfully, the sorted seller list does not change, and the sorted buyer group list becomes $\{B_1, ..., B_j' ..., B_{k-1}, B_k, ...\}$. B_{k-1} and b_k become a pair. As $B_{k-1} \ge B_k \ge b_k^s$, the final price for all buyer groups will be no less than B_{k-1}. Therefore, $p_i^{b'} \ge B_{k-1}/|g_j| \ge B_j/|g_j| \ge b_i^b = v_i^b$. Hence, $U_i^{b'} = v_i^b - p_i^{b'} \le 0 = U_i^b$.