

A large, intricate white scribble of overlapping lines on the left side of the teal cover, resembling a tangled web or a complex network.

logic

PAUL TOMASSI

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Logic

'Paul Tomassi's book is the most accessible and user-friendly introduction to formal logic currently available to students. Semantic and syntactic approaches are nicely integrated and the organisation is excellent, with later sections building systematically on earlier ones. Tomassi anticipates all the most important traps and confusions that students are likely to fall into and provides first-rate guidance on practical matters, such as strategies for proof-construction. Never intimidating, this is a text from which even the most unmathematically minded student can learn all the basics of elementary formal logic.'

E.J.Lowe, University of Durham

Logic brings elementary logic out of the academic darkness into the light of day and makes the subject fully accessible. Paul Tomassi writes in a patient and user-friendly style which makes both the nature and value of formal logic crystal clear. The reader is encouraged to develop critical and analytical skills and to achieve a mastery of all the most successful formal methods for logical analysis.

This textbook proceeds from a frank, informal introduction to fundamental logical notions, to a system of formal logic rooted in the best of our natural deductive reasoning in daily life. As the book develops, a comprehensive set of formal methods for distinguishing good arguments from bad is defined and discussed. In each and every case, methods are clearly explained and illustrated before being stated in formal terms. Extensive exercises enable the reader to understand and exploit the full range of techniques in elementary logic.

Logic will be valuable to anyone interested in sharpening their logical and analytical skills and particularly to any undergraduate who needs a patient and comprehensible introduction to what can otherwise be a daunting subject.

Paul Tomassi is a lecturer in Philosophy at the University of Aberdeen.

Logic

Paul Tomassi



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To Lindsey McLean,
Tiffin and Zebedee

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Preface

I felt compelled to write an introductory textbook about formal logic for a number of reasons, most of which are pedagogic. I began teaching formal logic to undergraduates at the University of Edinburgh in 1985 and have continued to teach formal logic to undergraduates ever since. Speaking frankly, I have always found teaching the subject to be a particularly rewarding pastime. That may sound odd. Formal logic is widely perceived to be a difficult subject and students can and often do experience problems with it. But the pleasure I have found in teaching the subject does not derive from the anxious moments which every student experiences to some extent when approaching a first course in formal logic. Rather, it derives from later moments when self-confidence and self-esteem take a significant hike as students (many of whom will always have found mathematics daunting) realise that they can manipulate symbols, construct logical proofs and reason effectively in formal terms. The educational value and indeed the personal pleasure which such an achievement brings to a person cannot be overestimated. Enabling students to take those steps forward in intellectual and personal development is the source of the pleasure I derive from teaching formal logic. In these terms, however, the problem with existing textbooks is that they generally make too little contribution to that end.

For example, each and every year during my time at Edinburgh the formal logic class contained a significant percentage of arts students with symbol-based anxieties. More worryingly, these often included intending honours students who had either delayed taking the compulsory logic course, failed the course in earlier years or converted to Philosophy late. Many of these students were very capable people who only needed to be taught at a gentler pace or to be given some individual attention. Moreover, even the best of those students who were not so daunted by symbols regularly got into difficulties simply through having missed classes—often for the best of reasons. Given the progressive nature of the formal logic course these students frequently just failed to catch up. As a teacher, it was immensely frustrating not to be able to refer students (particularly those in the final

category) to the textbook in any really useful way. The text we used was E.J.Lemmon's *Beginning Logic* [1965]. Undoubtedly, Lemmon's is, in many ways, an excellent text but the majority of students simply did not find it sufficiently accessible to be able to teach themselves from it. In all honesty, I think that this is quite generally the case with the vast majority of introductory texts in formal logic, i.e. inaccessibility is really only a matter of degree (albeit more so in the case of some than others). And this is no mere inconvenience for students and teachers. The underlying worry is that the consequent level of fail rates in formal logic courses might ultimately contribute to a decline in the teaching of formal logic in the universities or to a significant dilution of the content of such courses. For all of these reasons, I think it essential that we have a genuinely accessible introductory text which both covers the ground and caters to the whole spectrum of intending logic students, i.e. a text which enables students to teach themselves. That is what I have tried to produce here.

Logic covers the traditional syllabus in formal logic but in a way which may significantly reduce the kind of fail rates which, without such a text, are perhaps inevitable in compulsory courses in elementary logic offered within the Faculty of Arts. In the present climate, many faculties and, indeed, many philosophy departments consider such fail rates to be wholly unacceptable. Hence, the motivation to dilute the content of courses is obvious, e.g. by wholly omitting proof-theory. Personally, I believe that this cannot be a step in the right direction. In the last analysis, such a strategy either diminishes formal logic entirely or results in an unwelcome unevenness in the distribution of formal analytical skills among graduates from different institutions. I believe that the solution is to make available to students a genuinely accessible textbook on elementary logic which even the most anxious students in the class can use to teach themselves. Thus, *Logic* is not designed to promote my own view of formal logic as such or to promote the subject in any narrow sense. Rather, it is designed to promote formal logic in the widest sense, i.e. to make a subject which is generally perceived as difficult and inaccessible open and readily accessible to the widest possible audience.

To that end, the text is deliberately written in what I hope is a clear and user-friendly style. For example, formal statements of the rules of inference are postponed until the relevant natural deduction motivation has been outlined and an informal rule-statement has been specified. The text also makes extensive use of summary boxes of key points both during and at the end of chapters. Initial uses of key terms (and some timely reminders) are given in bold and such items are further explained in the glossary. Mock examination papers are also set at regular intervals in the text by way of dress rehearsal for the real thing. Given that accessibility is a crucial consideration, the pace of *Logic* is deliberately slow and indulgent. But this need not handicap either students or teachers. The text is exercise-intensive

and brighter students can simply move to more difficult exercises more quickly. Moreover, the very point of there being such a text is to enable students to teach themselves. So teachers need not move as slowly as the text, i.e. the pace of the course may very well be deliberately faster than that of the text. The point is that the text provides the necessary back-up for slower students anyway. Further, those who miss classes can plug gaps for themselves, and while I have no doubt that certain students will still have problems with formal logic the text is specifically designed to minimise the potential for anxiety attacks.

I should also add that the text is tried and tested at least in so far as a desktop version has been used successfully at the University of Aberdeen for the past three academic sessions, over which, as I write, class numbers have trebled. The success of the text is reflected as much in course evaluation responses as in the pass rate for Formal Logic 1 (only one student failed Formal Logic 1 over sessions 1994–5 and 1995–6). Further, the pass rate for the follow-on course, Formal Logic 2, was 100 per cent in the first academic session and 95 per cent in the second academic session. Despite the increase in class numbers, pass rates in both courses remain very high and the contents of course evaluation forms suitably reassuring.

A certain amount of motivation for writing *Logic* also stems from some unease not just about the style but about the content of existing textbooks. For although many excellent texts are available, there is something of an imbalance in most. For example, while a number of familiar texts are quite excellent on semantic methods these tend to be wholly devoid of (linear or Lemmon-style) proof-theory. In contrast, texts such as Lemmon, for example, show a clear bias towards proof-theory and are not as extensive in their treatment of semantic concepts and methods as they might be. Indeed, certain texts in this latter category are either devoid of semantic methods at the level of quantificational logic or devote a very limited amount of space to such topics. Yet another group of familiar texts involves rather less in the way of formal methods generally. Ultimately, I think, such texts include too little in that respect for purposes of teaching formal logic to undergraduates. Hence, there is a strong argument for an accessible textbook which strikes a fair balance between syntactic and semantic methods. To that end, *Logic* combines a comprehensive treatment of proof-theory not just with the truth-table method but also with the truth-tree method. After all, the latter method is quite mechanical throughout both propositional logic and the monadic fragment of quantificational logic. Moreover, if that method is given sufficient emphasis at an early stage students can also be enabled to apply the method beyond monadic quantificational logic. Of course, in virtue of undecidability with respect to invalidity at that level, there is no guarantee of the success of any purely mechanical application of the truth-tree method, i.e. infinite branches and infinite trees are possible. But the application of the method at that level, together with examples of infinite trees and

branches, vividly illustrates the consequences of undecidability to students and goes some way towards making clear just what is meant by undecidability. Finally, given that the method is also useful at the metatheoretical level, supplementing truth-tables with truth-trees from the outset seems a sound investment. In terms of content, then, the text covers the same amount of logical ground as any other text pitched at this level and, indeed, more than many.

In summary, *Logic* is primarily intended as a successful teaching book which students can use to teach themselves and which will enable even the most anxious students to grasp something of the nature of elementary logic. It is not intended to be a text which lecturers themselves will want to spend hours studying closely. Rather, it is intended to make a subject which is generally perceived as difficult and inaccessible open and easily accessible to the widest possible audience. In short, I hope that *Logic* constitutes a solution to what I believe to be a substantive teaching problem. However, if the text does no more than make formal logic accessible, comprehensible and above all useful to anxious students for whom it would otherwise have remained a mystery, then it will have fulfilled its purpose.

Paul Tomassi

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1

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1

How to Think Logically

I

Validity and Soundness

To study logic is to study **argument**. Argument is the stuff of logic. Above all, a logician is someone who worries about arguments. The arguments which logicians worry about come in all shapes and sizes, from every corner of the intellectual globe, and are not confined to any one particular topic. Arguments may be drawn from mathematics, science, religion, politics, philosophy or anything else for that matter. They may be about cats and dogs, right and wrong, the price of cheese, or the meaning of life, the universe and everything. All are equally of interest to the logician. Argument itself is the subject-matter of logic.

The central problem which worries the logician is just this: how, in general, can we tell good arguments from bad arguments? Modern logicians have a solution to this problem which is incredibly successful and enormously impressive. The modern logician's solution is the subject-matter of this book.

In daily life, of course, we do all argue. We are all familiar with arguments presented by people on television, at the dinner table, on the bus and so on. These arguments might be about politics, for example, or about more important matters such as football or pop music. In these cases, the term 'argument' often refers to heated shouting matches, escalating interpersonal altercations, which can result in doors being slammed and people not speaking to each other for a few days. But the logician is not interested in these aspects of argument, only in what was actually said. It is not the shouting but the sentences which were shouted which interest the logician.

For logical purposes, an *argument* simply consists of a sentence or a small set of sentences which lead up to, and might or might not justify, some other sentence. The division between the two is usually marked by a word such as 'therefore', 'so', 'hence' or 'thus'. In logical terms, the sentence or sentences leading up to the 'therefore'-type word are called **premises**. The sentence

which comes after the ‘therefore’ is the **conclusion**. For the logician, an *argument* is made up of premises, a ‘therefore’-type word, and a conclusion—and that’s all. In general, words like ‘therefore’, ‘so’, ‘hence’ and ‘thus’ usually signal that a conclusion is about to be stated, while words like ‘because’, ‘since’ and ‘for’ usually signal premises. Ordinarily, however, things are not always as obvious as this. Arguments in daily life are frequently rather messy, disordered affairs. Conclusions are sometimes stated before their premises, and identifying which sentences are premises and which sentence is the conclusion can take a little careful thought. However, the real problem for the logician is just how to tell whether or not the conclusion really does follow from the premises. In other words, when is the conclusion a **logical consequence** of the premises?

Again, in daily life we are all well aware that there are good, compelling, persuasive arguments which really do establish their conclusions and, in contrast, poor arguments which fail to establish their conclusions. For example, consider the following argument which purports to prove that a cheese sandwich is better than eternal happiness:

1. Nothing is better than eternal happiness.
 2. But a cheese sandwich is better than nothing.
- Therefore,
3. A cheese sandwich is better than eternal happiness.¹

Is this a good argument? Plainly not. In this case, the sentences leading up to the ‘therefore’, numbered ‘1’ and ‘2’ respectively, are the premises. The sentence which comes after the ‘therefore’, Sentence 3, is the conclusion. Now, the premises of this argument might well be true, but the conclusion is certainly false. The falsity of the conclusion is no doubt reflected by the fact that while many would be prepared to devote a lifetime to the acquisition of eternal happiness few would be prepared to devote a lifetime to the acquisition of a cheese sandwich. What is wrong with the argument is that the term ‘nothing’ used in the premises seems to be being used as a name, as if it were the name of some other thing which, while better than eternal happiness, is not quite as good as a cheese sandwich. But, of course, ‘nothing’ isn’t the name of anything.

In contrast, consider a rather different argument which I might construct in the process of selecting an album from my rather large record collection:

1. If it’s a Blind Lemon Jefferson album then it’s a blues album.
 2. It’s a Blind Lemon Jefferson album.
- Therefore,
3. It’s a Blues album.

Now, this argument is certainly a good argument. There is no misappropriation of terms here and the conclusion really does follow from the premises. In fact, both the premises and the conclusion are actually true; Blind Lemon Jefferson was indeed a bluesman who only ever made blues albums. Moreover, a little reflection quickly reveals that if the premises are true the conclusion must also be true. That is not to say that the conclusion is an eternal or **necessary truth**, i.e. a sentence which is always true, now and forever. But if the premises are actually true then the conclusion must also be actually true. In other words, this time, the conclusion really does follow from the premises. The conclusion is a logical consequence of the premises. Moreover, the necessity, the force of the 'must' here, belongs to the relation of consequence which holds between these sentences rather than to the conclusion which is consequent upon the premises. What we have discovered, then, is not the necessity of the consequent conclusion but the necessity of logical consequence itself.

In logical terms the Blind Lemon Jefferson argument is a **valid** argument, i.e. quite simply, if the premises are true, then the conclusion must be true, on pain of contradiction. And that is just what it means to say that an argument is valid: whenever the premises are true, the conclusion is guaranteed to be true. If an argument is valid then it is impossible that its premises be true and its conclusion false. Hence, logicians talk of validity as preserving truth, or speak of the transmission of truth from the premises to the conclusion. In a valid argument, true input guarantees true output.

Is the very first argument about eternal happiness and the cheese sandwich a valid argument? Plainly not. In that case, the premises were, indeed, true but the conclusion was obviously false. If an argument is valid then whenever the premises are true the conclusion is guaranteed to be true. Therefore, that argument is **invalid**. To show that an argument fails to preserve truth across the inference from premises to conclusion is precisely to show that the argument is invalid.

The Blind Lemon Jefferson example also illustrates the point that logic is not really concerned with particular matters of fact. Logic is not really about the way things actually are in the world. Rather, logic is about argument. So far as logic is concerned, Blind Lemon Jefferson might be a classical pianist, a punk rocker, a rapper, or a country and western artist, and the argument would still be valid. The point is simply that:

<i>If it's true that:</i>	If it's a Blind Lemon Jefferson album then it's a blues album.
<i>And it's true that:</i>	It's a Blind Lemon Jefferson album.
<i>Then it must be true that:</i>	It's a blues album.

However, if one or even all of the premises are false in actual fact it is still perfectly possible that the argument is valid. Remember: validity is simply

the property that if *the* premises are all true *then* the conclusion must be true. Validity is certainly not synonymous with truth. So, not every valid argument is going to be a good argument. If an argument is valid but has one or more false premises then the conclusion of the argument may well be a false sentence. In contrast, valid arguments with premises, which are all actually true sentences must also have conclusions which are actually true sentences. In Logicspeak, such arguments are known as **sound** arguments. Because a sound argument is a valid argument with true premises, the conclusion of every sound argument must be a true sentence. So, we have now discovered a very important criterion for identifying good arguments, i.e. *sound arguments are good arguments*. But surely we can say something even stronger here. Can't we simply say that sound arguments are definitely, indeed, definitively good arguments? Well, this is a controversial claim. After all, there are many blatantly circular arguments which are certainly sound but which are not so certainly good.

For example, consider the following argument:

1. Bill Clinton is the current President of the United States of America.
- Therefore,
2. Bill Clinton is the current President of the United States of America.

We can all agree that this argument is valid and, indeed, sound. But can we also agree that it is really a good argument? In truth, such arguments raise a number of questions some of which we will consider together later in this text and some of which lie beyond the scope of a humble introduction to what is ultimately a vast and variegated field of study. For present purposes, it is perfectly sufficient that you have a grasp of what is meant by saying that an argument is *valid* or *sound*.

To recap, sound arguments are valid arguments with true premises. A valid argument is an argument such that if the premises are true then the conclusion must be true. Hence, the conclusion of any sound argument must be true. But do note carefully that validity is not the same thing as truth. Validity is a property of arguments. Truth is a property of individual sentences. Moreover, not every valid argument is a sound argument. Remember: a valid argument is simply an argument such that if the premises are true then the conclusion must be true. It follows that arguments with one or more premises which are in fact false and conclusions which are also false might still be valid none the less. In such cases the logician still speaks of the conclusion as being validly drawn even if it is false. On false conclusions in general, one American logician, Roger C. Lyndon, prefaces his logic text with the following quotation from Shakespeare's *Twelfth Night*: 'A false conclusion; I hate it as an unfilled can.'² That sentiment is no doubt particularly apt as regards a false

conclusion which is validly drawn. None the less, it is perfectly possible for a false conclusion to be validly drawn. For example:

1. If I do no work then I will pass my logic exam.
 2. I will do no work.
- Therefore,
3. I will pass my logic exam.

So, not all valid arguments are good arguments, but the important point is that even though the conclusion is false, the argument is still valid, i.e. if its premises really were true then its conclusion would also have to be true. Hence, the conclusion is validly drawn from the premises even though the conclusion is false.

Moreover, valid arguments with false premises can also have actually true conclusions. For example:

1. My uncle's cat is a reptile.
 2. All reptiles are cute, furry creatures.
- Therefore,
3. My uncle's cat is a cute, furry creature.

This time both premises are false but the conclusion is true. Again, the argument is valid none the less, i.e. it is still not possible for the conclusion to be false if the premises are true. Further, while we might not want to say that this particular argument is a good one, it is worth pointing out that there are ways in which we can draw conclusions from a certain kind of false sentence which leads to a whole class of arguments which are obviously good arguments. We will consider just this kind of reasoning in some detail later in Chapter 3. For now, remember that validity is not synonymous with truth and that validity itself offers no guarantee of truth. If the premises of a valid argument are true then, certainly, the conclusion of that argument must be true. But just as a valid argument may have true premises, it may just as easily have false premises or a mixture of both true and false premises. Indeed, valid arguments may have any mix of true or false premises with a true or false conclusion excepting only that combination of true premises and false conclusion. Only sound arguments need have actually true premises and actually true conclusions. Therefore, soundness of argument is the criterion which takes us closest to capturing our intuitive notion of a good argument which genuinely does establish its conclusion. Whether we can simply identify soundness of argument with that intuitive notion of good argument remains controversial. But

what is surely uncontroversial is that validity and soundness of argument are integral parts of any attempt to make that intuition clear.

II Deduction and Induction

In the ordinary business of daily life (and particularly in films about Sherlock Holmes) we generally find the term 'deduction' used in a very loose sense to describe the process of reasoning from a set of premises to a conclusion. In contrast, logicians tend to use the same term in a rather narrower sense. For the logician, **deductive argument** is valid argument, i.e. validity is the logical standard of deductive argument. Hence, you will frequently find valid arguments referred to as *deductively valid* arguments.

In Logicspeak the premises of a valid argument are said to **entail** or **imply** their conclusion and that conclusion is said to be *deducible* from those premises. But deduction is not the only kind of reasoning recognised by logicians and philosophers. Rather, deduction is one of a pair of contrasting kinds of reasoning. The contrast here is with **induction** and **inductive argument**. Traditionally, while deduction is just that kind of reasoning associated with logic, mathematics and Sherlock Holmes, induction is considered to be the hallmark of scientific reasoning, the hallmark of scientific method. For the logician deductive reasoning is valid reasoning. Therefore, if the premises of a deductive argument are true then the conclusion of that argument must be true, i.e. validity is truth-preserving. But validity is certainly not the same as truth and deduction is not really concerned with particular matters of fact or with the way things actually are in the world. In sharp contrast, and just as we might expect of scientists, induction is very much concerned with the way things actually are in the world.

We can see this point illustrated in one rather simple kind of inductive argument which involves reasoning, as we might put it, from the particular to the general. Such arguments proceed from a set of premises reporting a particular property of some specific individuals to a conclusion which ascribes that property to every individual, quite generally. Inductive arguments of this kind proceed, then, from premises which need be no more than records of personal experience, i.e. from *observation-statements*. These are **singular sentences** in the sense that they concern some particular individual, fact or event which has actually been observed. For example, suppose you were acquainted with ten enthusiastic and very industrious logic students. You might number these students 1, 2, 3 and so on and proceed to draw up a list of premises as follows:

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